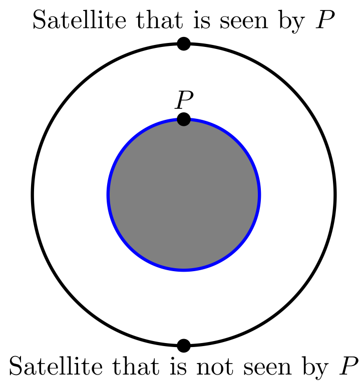
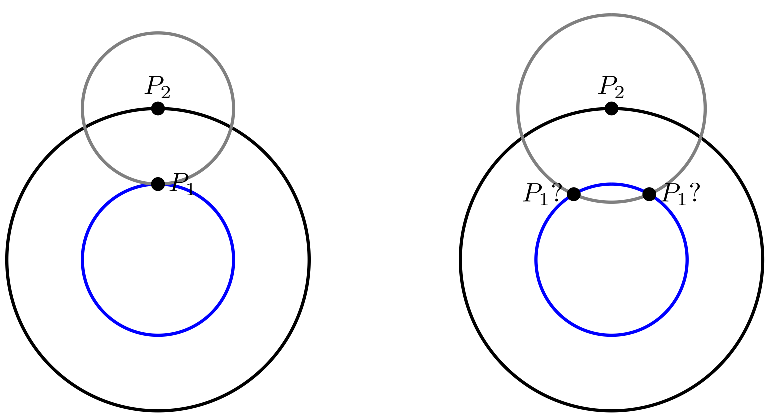
planarPositioning Solution - MathGames

June 11, 2024

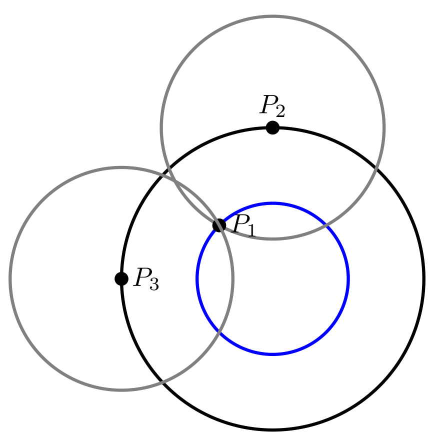
Define circle c_1 as the surface of the planet and circle c_2 as the circle in which the satellites fly. If the line segment between a point P on the planet and a satellite does not pass through the planet, then in this solution it will be said that P sees that satellite.



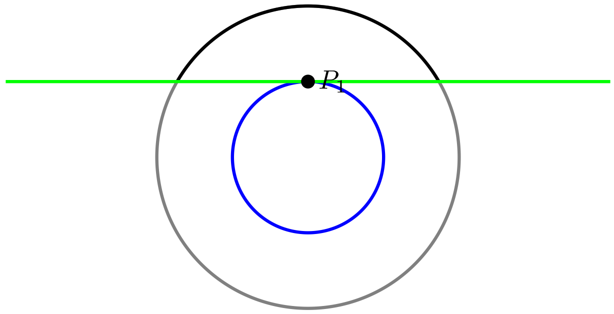
If a point P_1 sees exactly one satellite P_2, then the possible positions for point P_1 is the intersection between the planet and a circle with middle point P_2 and radius $d(P_1, P_2)$. The location of point P_1 is only known if point P_1 lies directly below the satellite. Otherwise, there are two intersection points.



If a point P_1 sees exactly two satellites, P_2 and P_3, then the location of P_1 is known, unless the two satellites are exactly opposite of eachother. However, it is not possible to simultaneously see two opposite satellites.



A point P_1 sees a satellite if and only if the satellite is at or above the tangent line of the planet through point P. The arc of points in c_2 that satisfy this property will be referred to as l.



in order to ensure that the GPS system works for every point, one must ensure that the arc length between the satellites is less than or equal to halve the length of l. The length of l can be calculated with $R \cdot \cos^{-1} R/r$, where R is the radius of c_2 and r is the radius of c_1. Therefor, the answer is $\lceil \frac{2 \cdot \pi \cdot R}{R \cdot \cos^{-1} R/r} \rceil$.